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Question Paper Code: 41310

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Third Semester Civil Engineering

MA 6351 – TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS (Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Computer and Communication Engineering/

Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/ Instrumentation and Control Engineering/Manufacturing Engineering/Marine

Engineering/Materials Science and Engineering/Mechanical Engineering/ Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation Engineering/Bio Technology/Chemical Engineering/Chemical and Electrochemical Engineering/Food Technology/Information Technology/

Petrochemical Technology/Petroleum Engineering/Plastic Technology/

Polymer Technology) (Regulations 2013)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

- 1. Find the complete integral of the PDE : $z = px + qy + \sqrt{pq}$
- 2. Solve: $(D^3 3DD^{\prime 2} + 2D^{\prime 3}) z = 0$
- 3. Find b_n in the expansion of $f(x) = x^2$ as a Fourier series in $(-\pi, \pi)$.
- 4. Define Root mean square value of a function.
- 5. Classify the partial differential equation $u_{xy} = u_x u_y + xy$.
- 6. State possible solutions of the one dimensional heat equation.
- 7. Find the Fourier transform of $f(x) = \begin{cases} 1 & \text{in } |x| < a \\ 0 & \text{in } |x| > a \end{cases}$



- 8. State the convolution theorem for Fourier transforms.
- 9. Find the Z-transform of {n}.
- 10. Prove that $Z\{nf(n)\} = -z \frac{d}{dz} F(z)$, where $Z\{f(n)\} = F(z)$.

PART - B

(5×16=80 Marks)

- 11. a) i) Find the singular solution of the equation $z = px + qy + p^2 + pq + q^2$. (8)
- ii) Solve: x(y z) p + y(z x) q = z(x y). (OR)
 - b) i) Solve: $(D^2 + 4DD' 5D'^2)z = \sin(x 2y) + e^{2x y}$. (8)
- ii) Solve: $(D^2 + DD' 6D'^2)z = y\cos x$. (8)
- 12. a) i) Find the Fourier series for $f(x) = x^2 in \pi < x < \pi$. (8)
 - ii) Find the half range cosine series for $f(x) = x (\pi x)$ in $(0, \pi)$. (8)
 - b) Find the Fourier series expansion upto the first three harmonics for the function defined in the following table (16)

х	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2 π
y	1	1.4	1.9	1.7	1.5	1.2	1.0

13. a) A string is stretched and fastened to two points x = 0 and x = l apart. Motion is started by displacing the string into the form $y = k(lx - x^2)$ from which it is released at time t = 0. Find the displacement of any point on the string at a distance of x from one end at time t. (16)

(OR)

b) A bar, 10 cm long with insulated sides, has its ends A and B kept at 20°C and 40°C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50°C and at the same instant that at B is lowered to 10°C. Find the subsequent temperature at any point of the bar at any time. (16)



14. a) Find the Fourier transform of the function f(x) defined by $f(x) = \begin{cases} 1-x^2 & \text{in } |x| \le 1 \\ 0 & \text{in } |x| > 1 \end{cases}$.

Hence prove that $\int_0^\infty \frac{\sin s - s \cos s}{s^3} \cos \left(\frac{s}{2}\right) ds = \frac{3\pi}{16}$. Also show that

$$\int_0^\infty \frac{(x \cos x - \sin x)^2}{x^6} dx = \frac{\pi}{15}.$$
 (16)

(OR)

- b) i) Show that the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$ is $e^{\frac{-e^2}{2}}$. (8)
 - ii) Find the Fourier cosine transform of $e^{-a^2x^2}$ and hence find the Fourier sine transform of $xe^{-a^2x^2}$. (8)
- 15. a) i) Find the inverse Z-transform of $\frac{8z^2}{(2z-1)(4z+1)}$ using convolution theorem for Z-transforms. (8)
 - ii) Find the inverse Z-transform of $\frac{z^2-3z}{(z-5)(z+2)}$ using residue theorem. (8)
 - b) i) Solve: $y_{n+2} 4y_{n+1} + 4y_n = 0$, $y_0 = 1$, $y_1 = 0$, using Z-transform. (10)
 - ii) Find the Z-transform of $\{n\}$ and $\left\{\frac{1}{n+1}\right\}$. (6)

14. a) Find the Faurier transform of the function fits defined by $f(x) = \begin{cases} 1-x^2 & \text{in } |x| \le 1 \\ 0 & \text{in } |x| \le 1 \end{cases}$

Hence prove that $\int_{1}^{\infty} \frac{\sin x - \cos x}{x^2} \cos \left(\frac{x}{x}\right) dx = \frac{1\pi}{16}$. Also show that

$$\frac{\partial I}{\partial x} = x B \frac{dx}{(x \cdot (x) - x \cdot x \cdot \cos x)} = 1$$

(DIO)

- (6) i) Show that the Fourier transform of Opp = n⁻¹ is n⁻¹. (8).
- that the Fourier cosine transform of e ' and honce find the Fourier size transform of xe. (8)
 - 16. at 11 Find the inverse 2-transform of $\frac{8x^2}{(2a-1)(4x+1)}$ using convolution theorem

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- if Find the inverse 2-transform of $\frac{x^2-3\pi}{(x-5)(x+2)}$ using residue-theorem (8)
- (10) If Solve: $y_{med} 4y_{med} + 4y_m = 0$, $y_0 = 1$, $y_1 = 0$, using Z-trainsform. (10)
- ii) Find the 2-transform of (a) and $\left\{\frac{1}{n+1}\right\}$. (6)